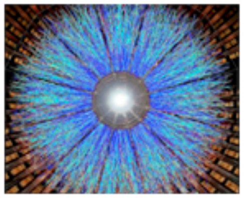
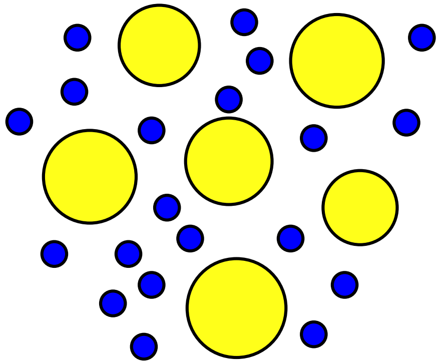
School of Collective Dynamics in High Energy Collisions

June 7 - 11, 2010



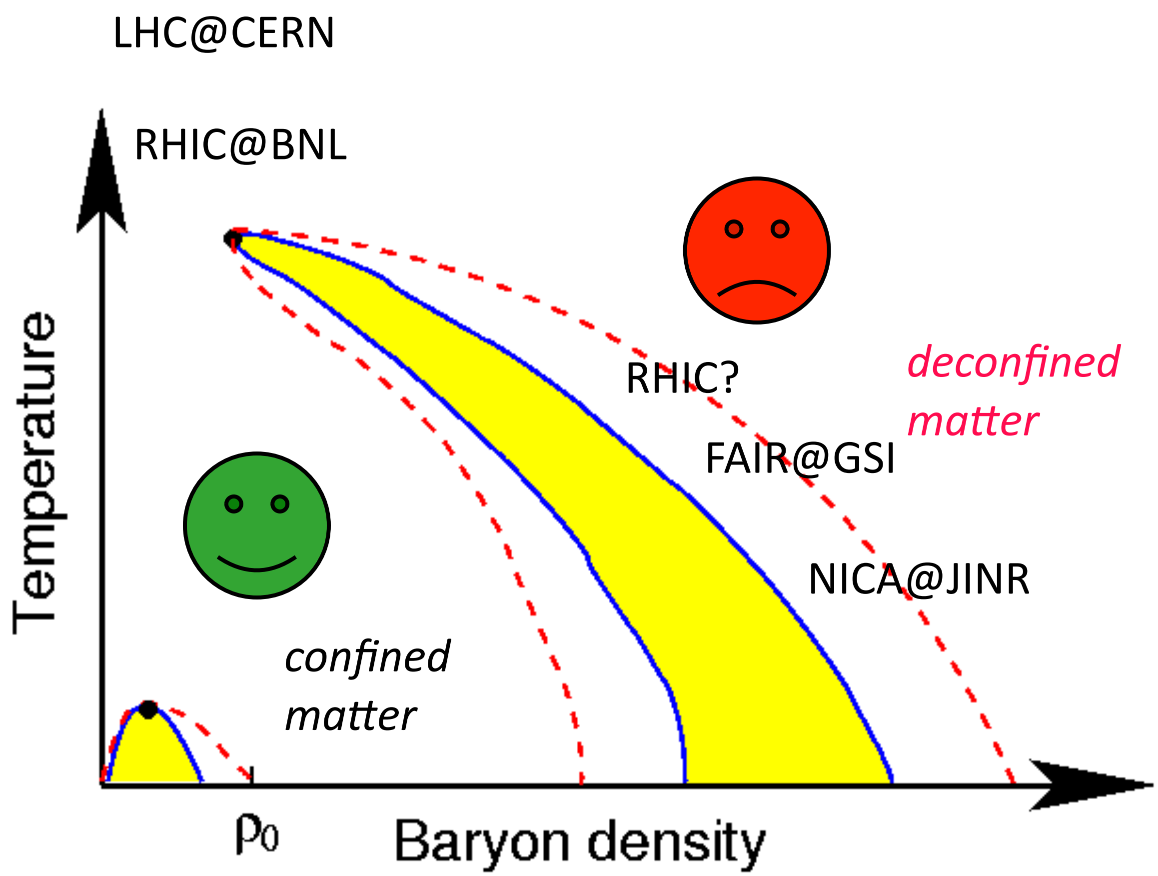
Nuclear liquid-gas phase transition

Jørgen Randrup



Schematic and simplified

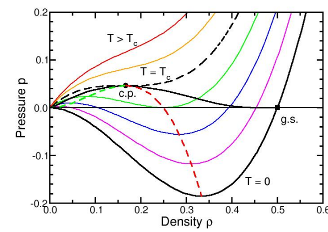
phase diagram of strongly interacting matter



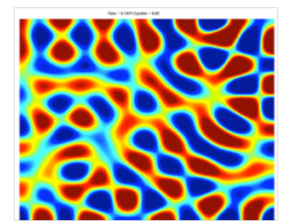
The nuclear liquid-gas phase transition

revealed by collective dynamics in energetic nuclear collisions

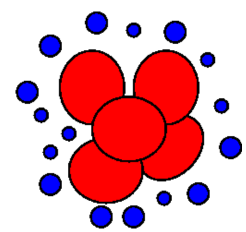
- Thermodynamics: Phase coexistence



- Spinodal instability: Dispersion relations

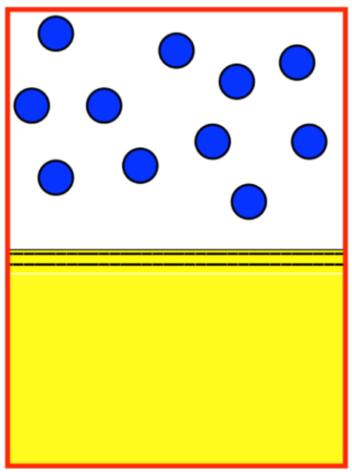


- Transport simulation: Spinodal fragmentation



Nuclear liquid-gas phase coexistence

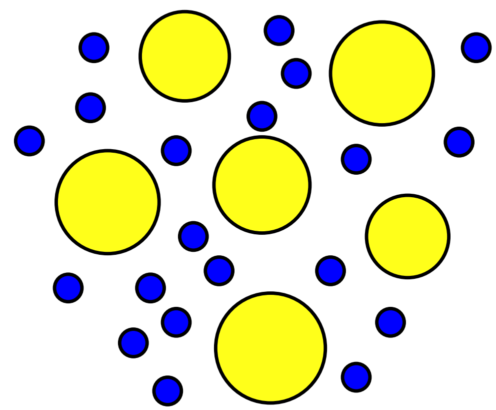
nucleon gas phase



nuclear liquid phase
(nuclear matter)

can coexist in mutual equilibrium

$\neq$

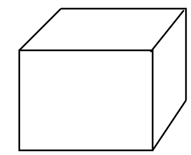


phase mixture

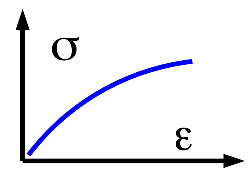
Thermodynamics (no conserved charges):

Statistical equilibrium in bulk matter

Control parameter(s) $\{X\}$: Energy $E = V\varepsilon$; Volume $V \to \infty$



Entropy function $S\{X\}$: $S(E,V) = V\sigma(\varepsilon)$



Derivative(s) $\lambda_X = \partial_X S$:

$\beta = 1/T = \partial_E S(E,V) = \partial_\varepsilon \sigma(\varepsilon)$ *temperature*

$\pi = p/T = \partial_V S(E,V) = \sigma - \beta\varepsilon$ *pressure*

Thermodynamic *coexistence*: $T_1 = T_2$ & $p_1 = p_2$

$\delta S_{tot} = 0 \Rightarrow (\partial_\chi \sigma)_1 = (\partial_\chi \sigma)_2$ <=> $\sigma(\varepsilon)$ *has common tangent!*

#1

Thermodynamic (local) *stability*: $\delta^2 S_{tot} < 0$

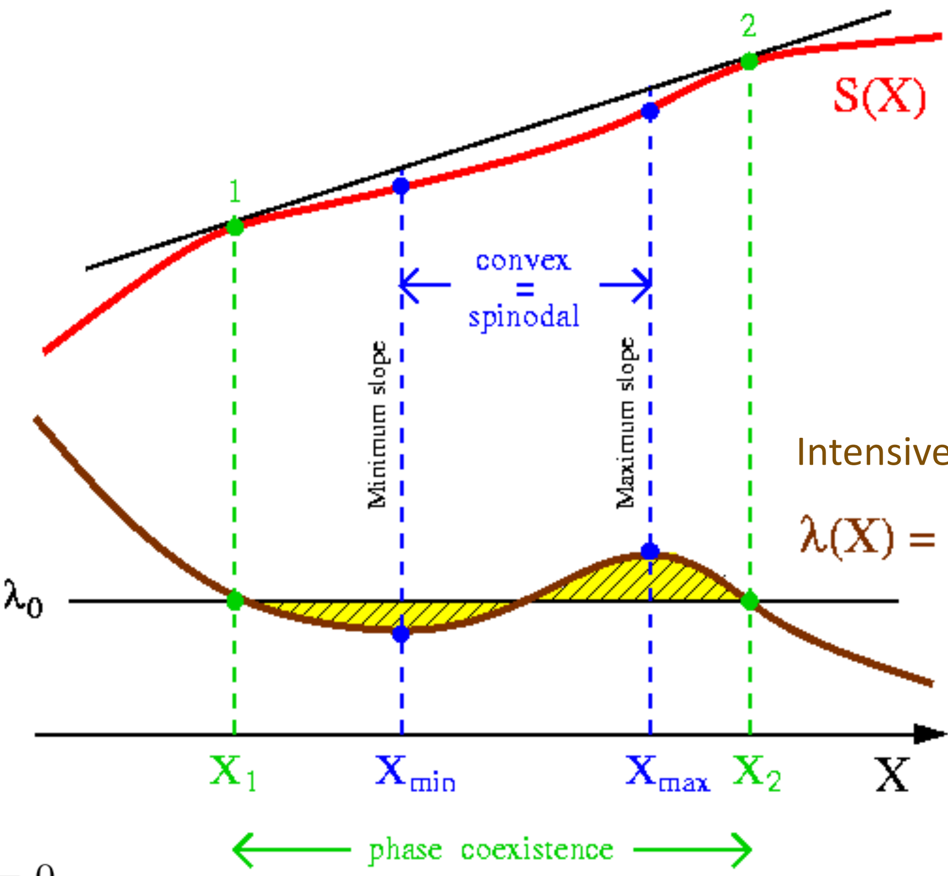
=> Curvature matrix $\{\partial_\chi \partial_{\chi'} \sigma\}$ has only *negative* eigenvalues

First order <=> Phase coexistence <=> Spinodal instability

Extensive variable X

Entropy function S(X)

… occur when S(X) is locally convex:



$p = T\sigma - \varepsilon$

Intensive variable:

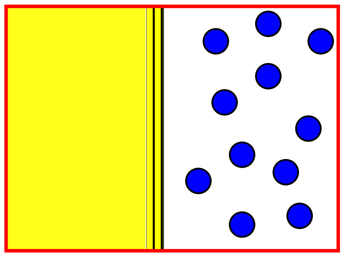
$\lambda(X) = -dS/dX$

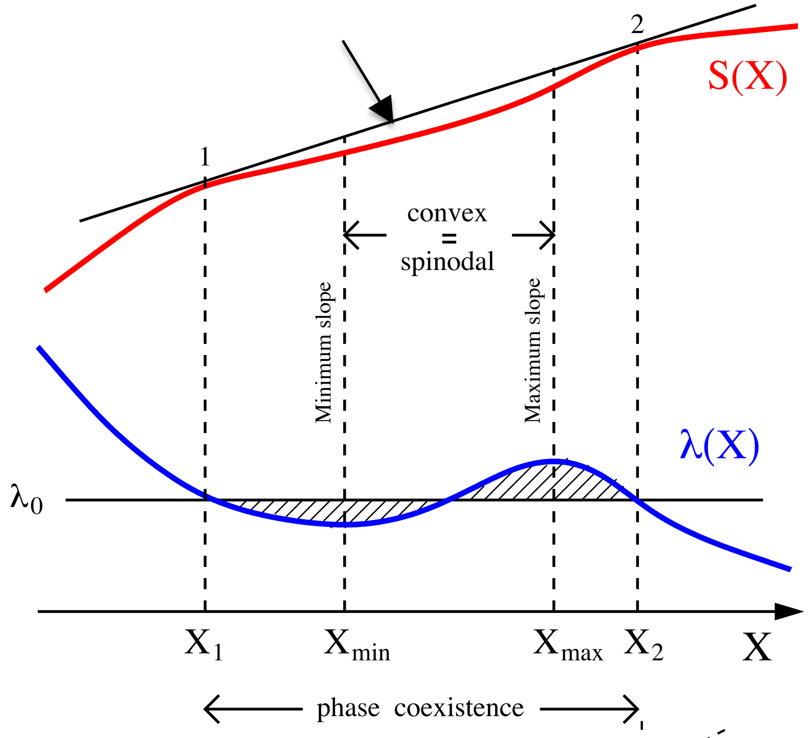
[X=E => $\lambda=\beta$]

Maxwell construction:

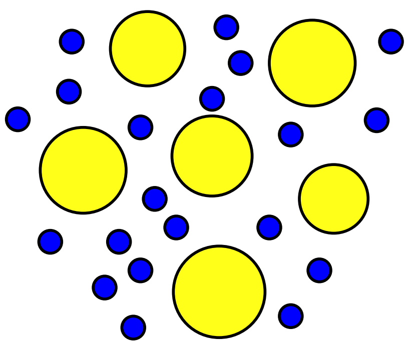
$$\int_{X_1}^{X_2} dX\,(\lambda(X) - \lambda_0) = 0$$

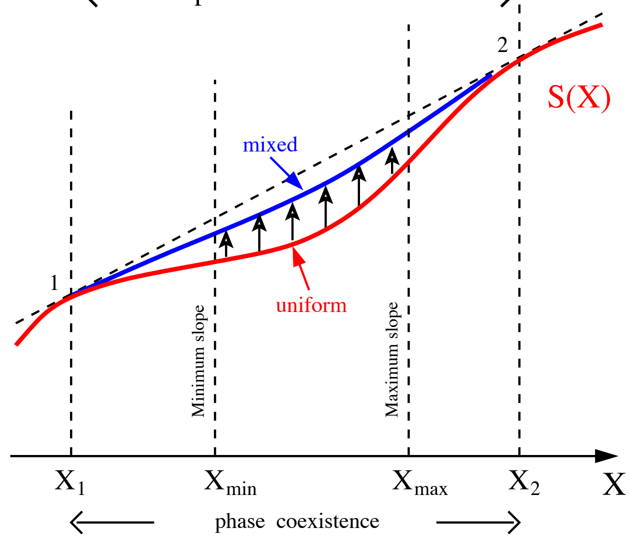
Separated phases:





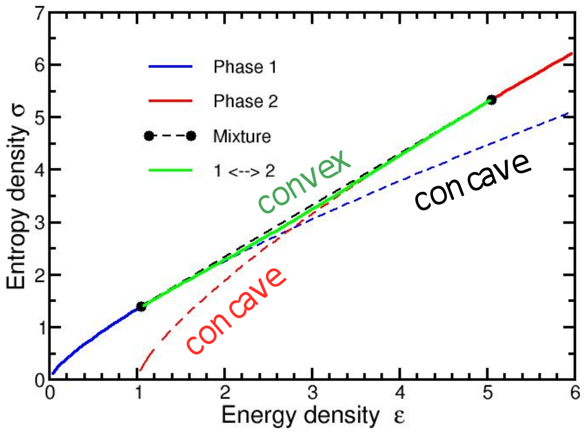
Mixed phases:



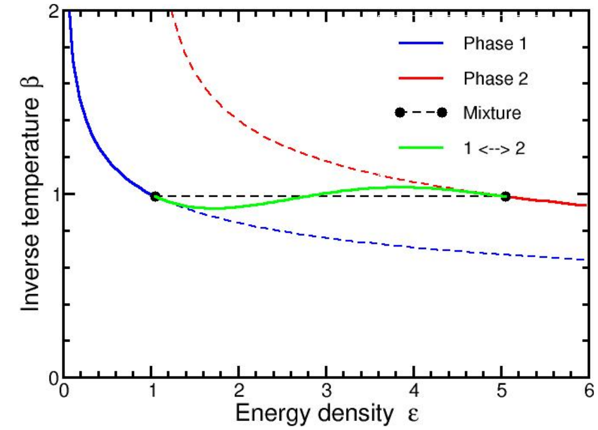


Simplest example: No conserved charges

Entropy density: $\sigma(\varepsilon)$

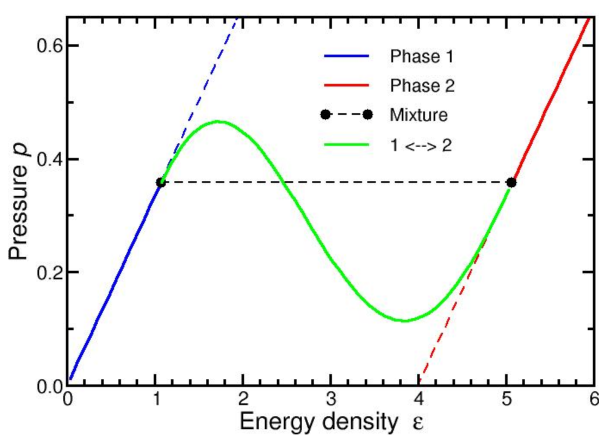


Inverse temperature: $\beta(\varepsilon) = \partial\sigma/\partial\varepsilon$

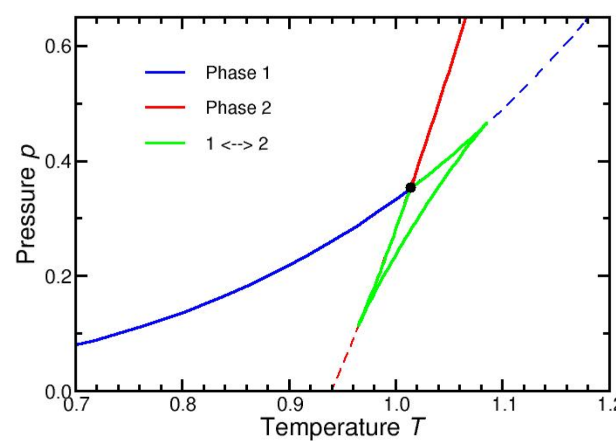


Equation of State

Pressure: $p(\varepsilon) = T\sigma - \varepsilon$

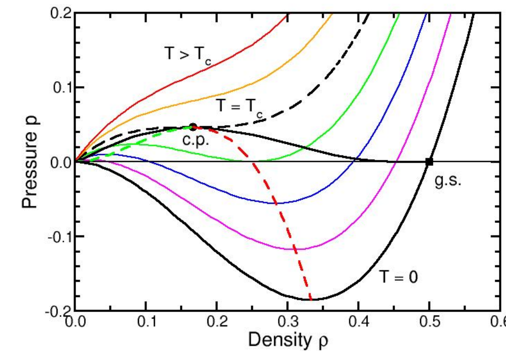


Pressure: $p(T)$



Thermodynamics (one charge):

Statistical equilibrium in bulk matter



Control parameter(s) $\{X\}$:

- Energy $E = V\varepsilon$
- Number $N = V\rho$
- Volume $V \to \infty$

Entropy function $S\{X\}$:

$$S(E,N,V) = V\sigma(\varepsilon,\rho)$$

Derivative(s) $\lambda_X = \partial_X S$:

$$\beta = 1/T \;=\; \partial_E S(E,N,V) \;=\; \partial_\varepsilon \sigma(\varepsilon,\rho)$$

$$\alpha = -\mu/T = \partial_N S(E,N,V) = \partial_\rho \sigma(\varepsilon,\rho)$$

$$\pi = p/T \;=\; \partial_V S(E,N,V) = \sigma - \beta\varepsilon - \alpha\rho$$

Thermodynamic *coexistence*:
$\delta S_{tot} = 0 \;\Rightarrow\; (\partial_\chi \sigma)_1 = (\partial_\chi \sigma)_2$

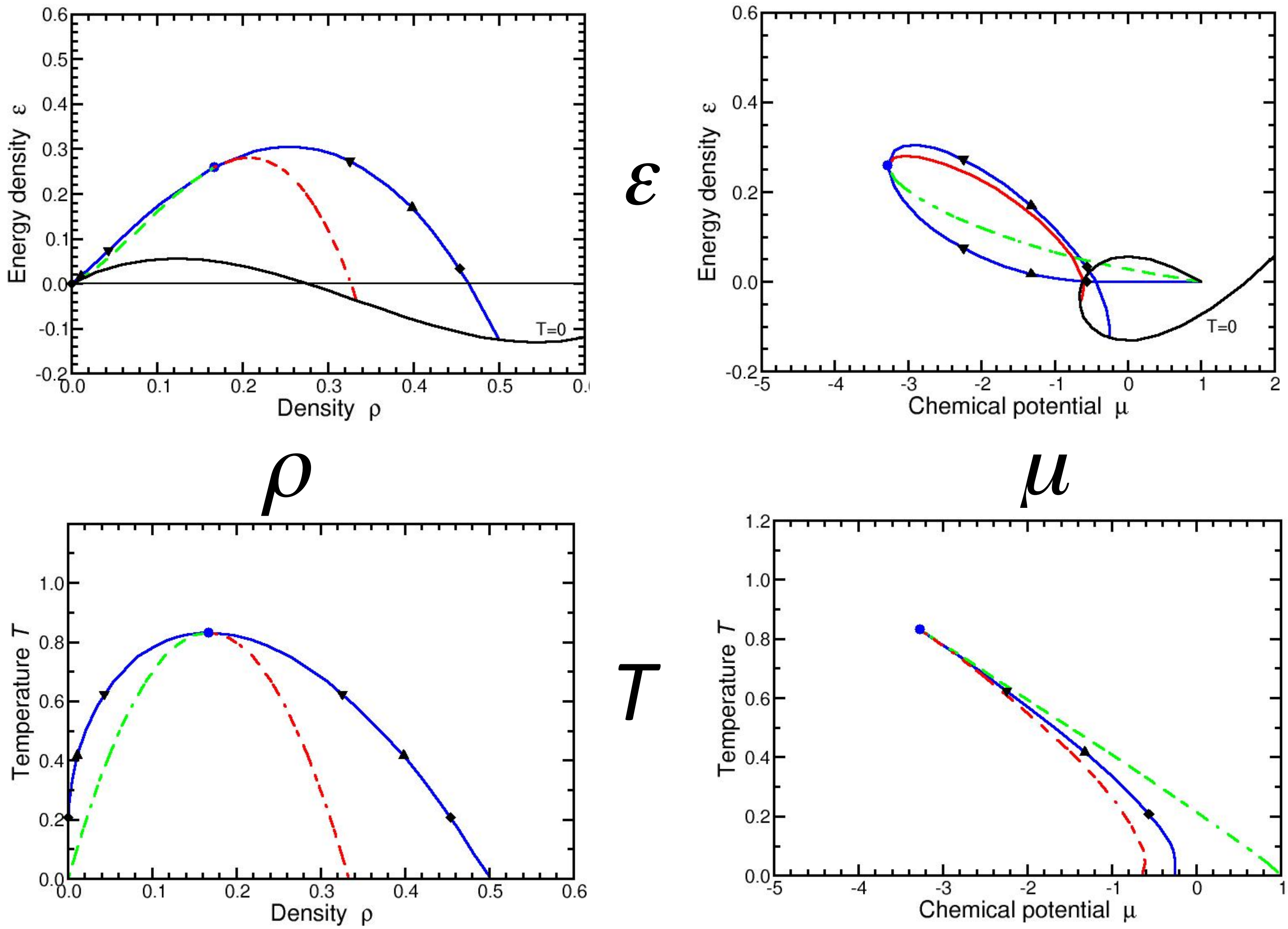
$T_1 = T_2$ & $\mu_1 = \mu_2$ & $p_1 = p_2$

<=> $\sigma(\varepsilon,\rho)$ *has common tangent!*

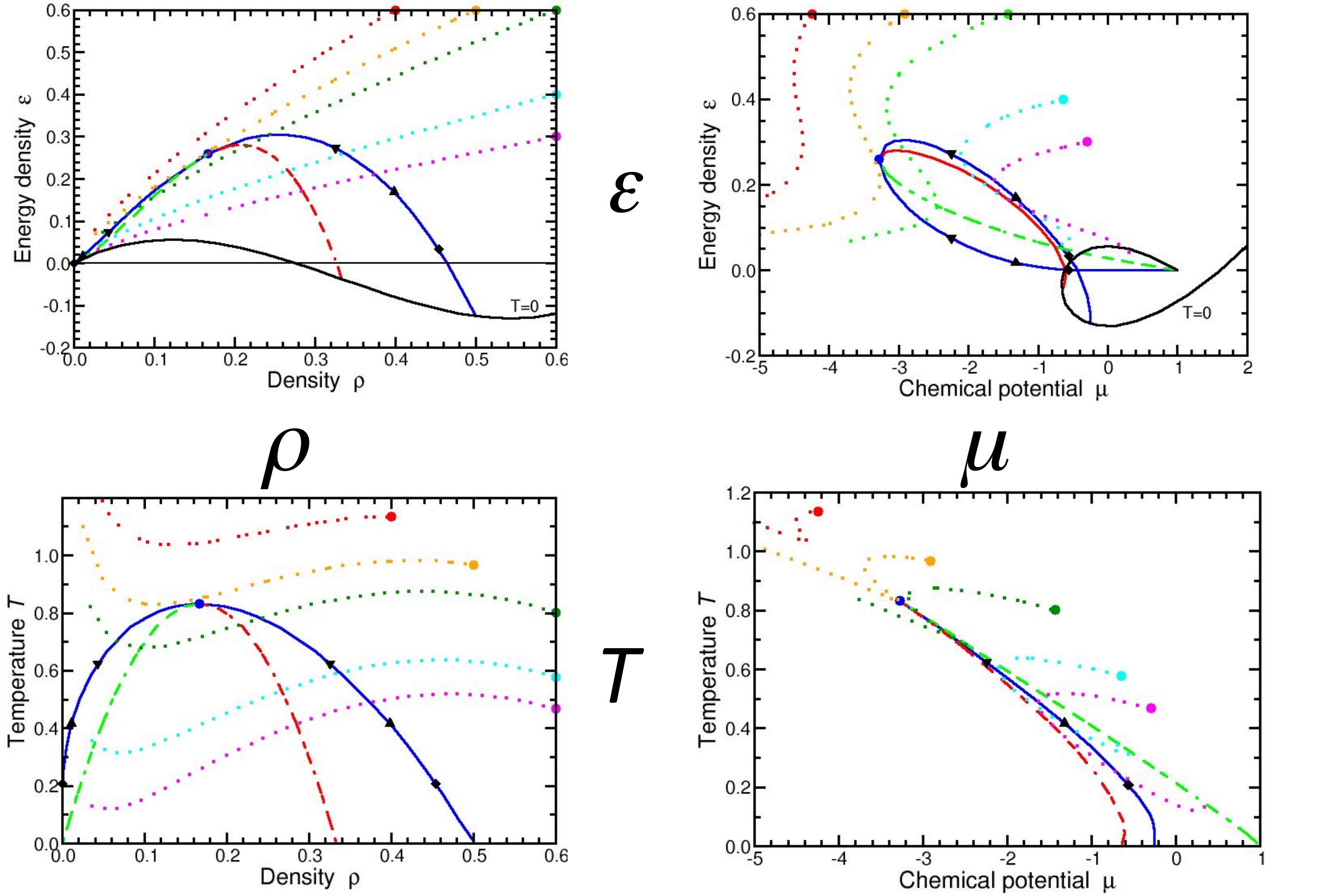
#2

Thermodynamic (local) *stability*: $\delta^2 S_{tot} < 0$
=> Curvature matrix $\{\partial_\chi \partial_{\chi'} \sigma\}$ has only *negative* eigenvalues

Nuclear phase diagram in different representations



Isentropic phase trajectories in different representations



Microcanonical -> Canonical:

entropy density $\sigma(\varepsilon,\rho)$ $\Rightarrow \begin{cases} \beta(\varepsilon,\rho) = \partial_\varepsilon \sigma(\varepsilon,\rho) = 1/T(\varepsilon,\rho) \\ \alpha(\varepsilon,\rho) = \partial_\rho \sigma(\varepsilon,\rho) = -\mu(\varepsilon,\rho)/T(\varepsilon,\rho) \end{cases}$

temperature

chemical potential

$$\Rightarrow \begin{cases} p(\varepsilon,\rho) = \sigma T - \varepsilon + \mu\rho \\ h(\varepsilon,\rho) = p + \varepsilon \end{cases}$$

pressure

enthalpy density

Canonical scenario: specified temperature T

free energy density

$$f_T(\rho) \;\equiv\; \varepsilon_T(\rho) - T\sigma_T(\rho) \;=\; \mu_T(\rho)\rho - p_T(\rho)$$

$$\mu_T(\rho) = \partial_\rho f_T(\rho)$$

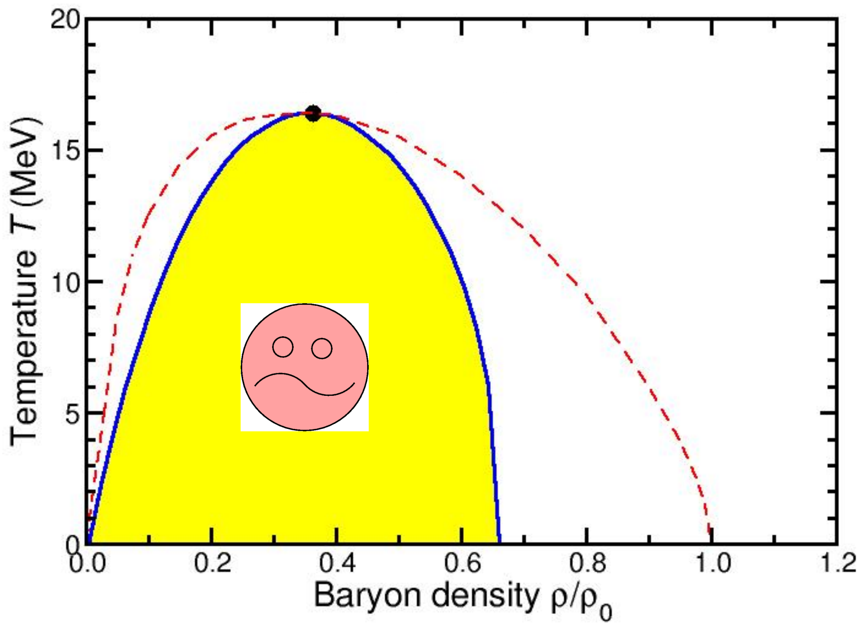
$$\sigma_T(\rho) = -\partial_T f_T(\rho)$$

#3

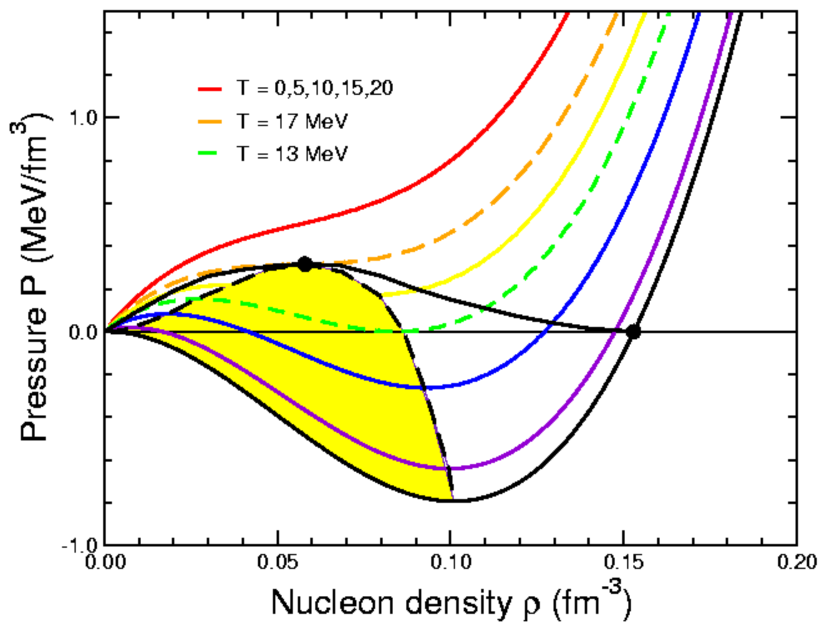
$f_T(\rho)$ has common tangent!

Nuclear matter

Phase diagram



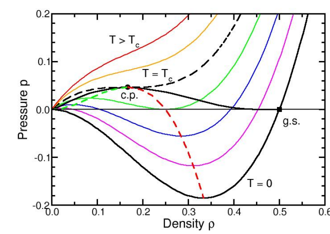
Equation of state: $p_T(\rho)$



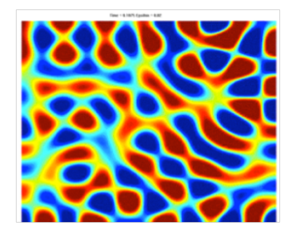
The nuclear liquid-gas phase transition

revealed by collective dynamics in energetic nuclear collisions

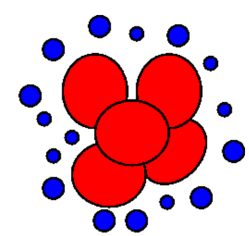
✓ • Thermodynamics: Phase coexistence



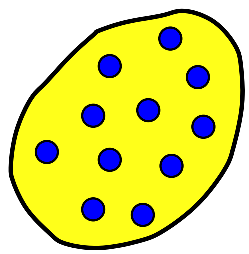
• Spinodal instability: Dispersion relations



• Transport simulation: Spinodal fragmentation

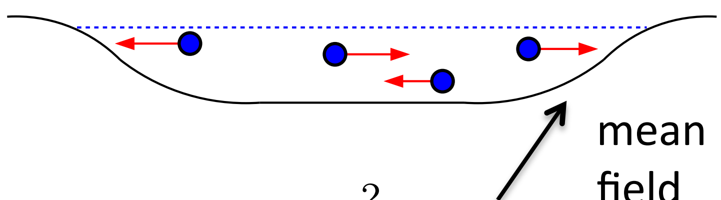


Nuclear dynamics at $E_{coll} \approx E_{Fermi}$



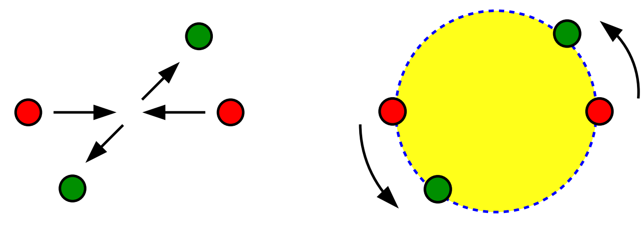
Individual nucleons move in common one-body field while occasionally experiencing Pauli-suppressed binary collisions

One-particle Hamiltonian



$$h[f](r,p) \;=\; \frac{p^2}{2m} + U[\rho](r)$$

Two-body collisions



position space

momentum space

The state of the system is characterized by its reduced one-particle phase-space density:

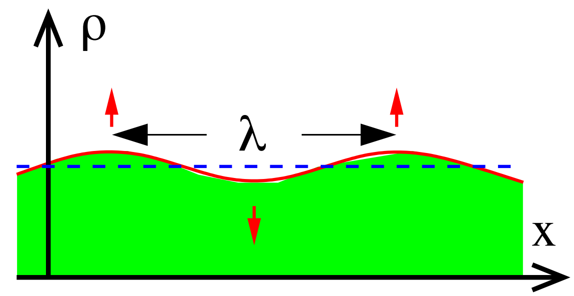
$$f(r,p)$$

Collective modes (sound waves)

Consider a small harmonic distortion:

$$\rho(r,t) \doteq \rho_0 + \delta\rho(r,t)$$

$$\delta\rho(r,t) \sim \mathrm{e}^{ikx - i\omega_k t}$$



Use equations of motion to get ω(k) (``dispersion relation'')

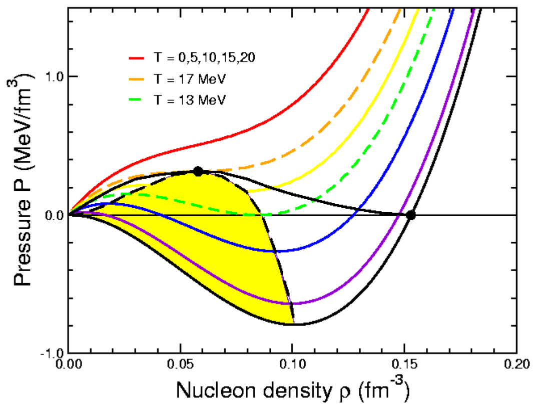
$$\omega_k = \epsilon_k + i\gamma_k$$

Inside the spinodal region Re[ω_k]=0, so $\omega_k = i\gamma_k$

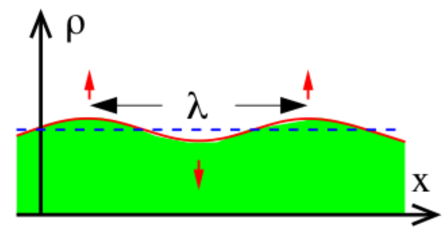
$$\delta\rho(r,t) \sim \mathrm{e}^{ikx + \gamma_k t}$$

Exponential amplification

Spinodal pattern formation

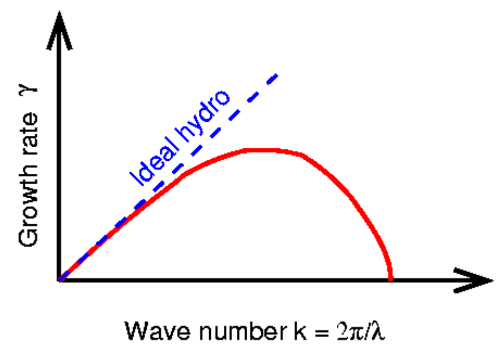


Density undulations are amplified in the spinodal region:

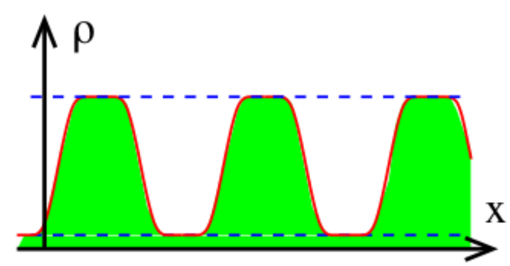


Long-wavelength distortions grow slowly (it takes time to relocate the matter)

Short-wavelength distortions grow slowly (they are hardly felt due to finite range)



There is an *optimal length scale* that grows faster than all others



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263

Spinodal decomposition occurs in many areas of science and technology, for example:

Colloidal aggregation

Polymer blends

Binary fluid mixtures

Binary metallic alloys

Inorganic glasses

Sticky emulsions

It also plays a role in nuclear *multifragmentation*!

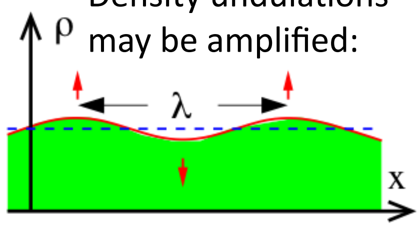
Does it play a role in the *hadronization* of the QGP?

Nuclear spinodal instabilities

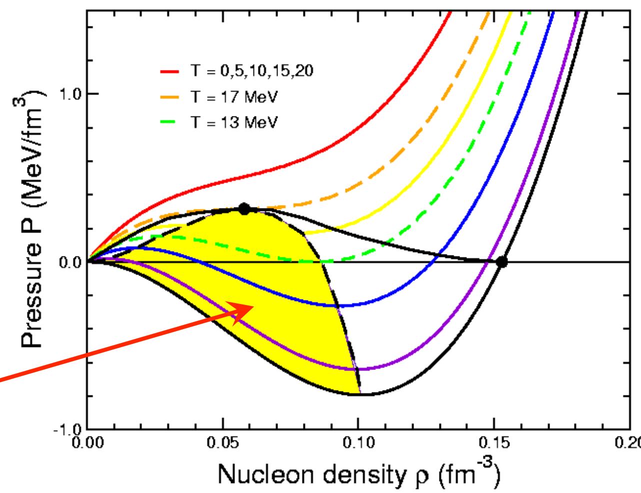
Spinodal region: $F_0 < -1$
Matter is thermodynamically and mechanically unstable

$$F_0 = \frac{\partial h}{\partial \epsilon_F}$$

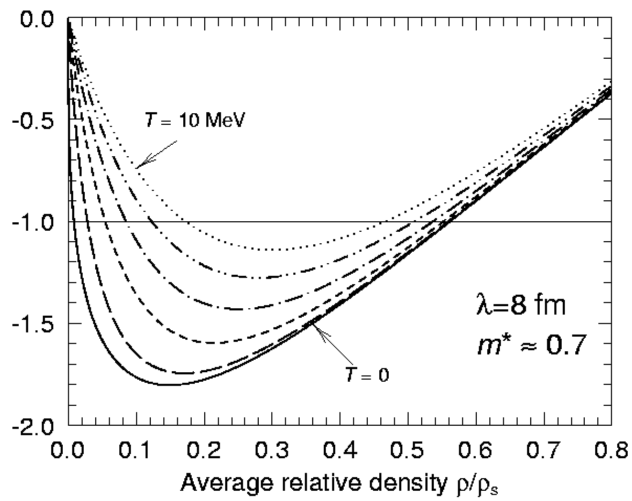
Density undulations may be amplified:

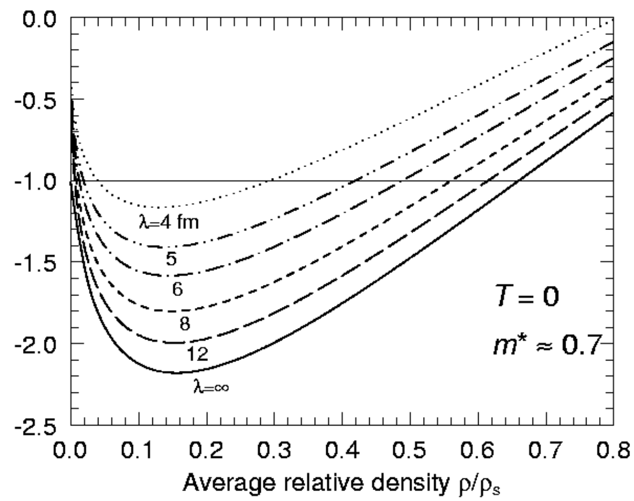


Nuclear Matter Equation of State:



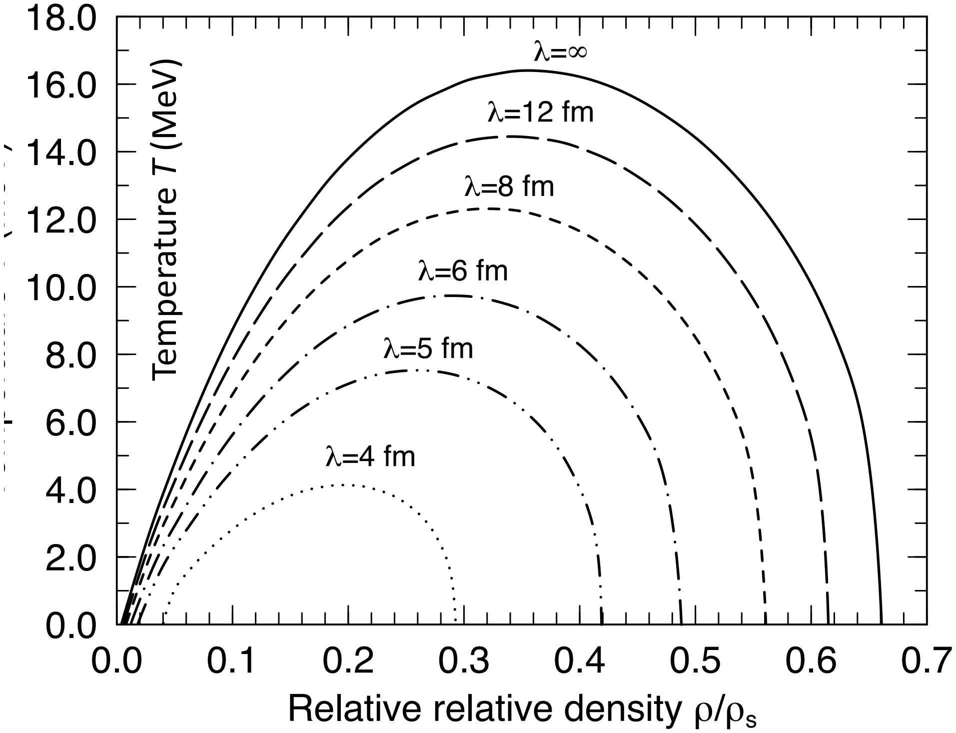
The Landau parameter F_0 depends on ρ, T, λ:



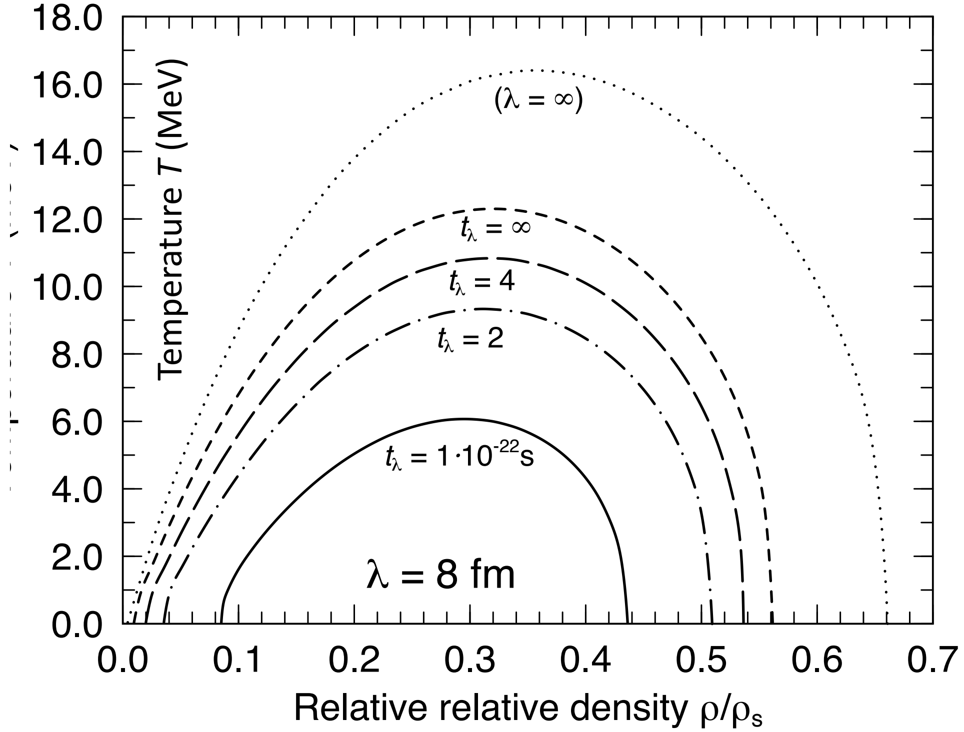


Spinodal boundaries in the (ρ,T) phase plane:

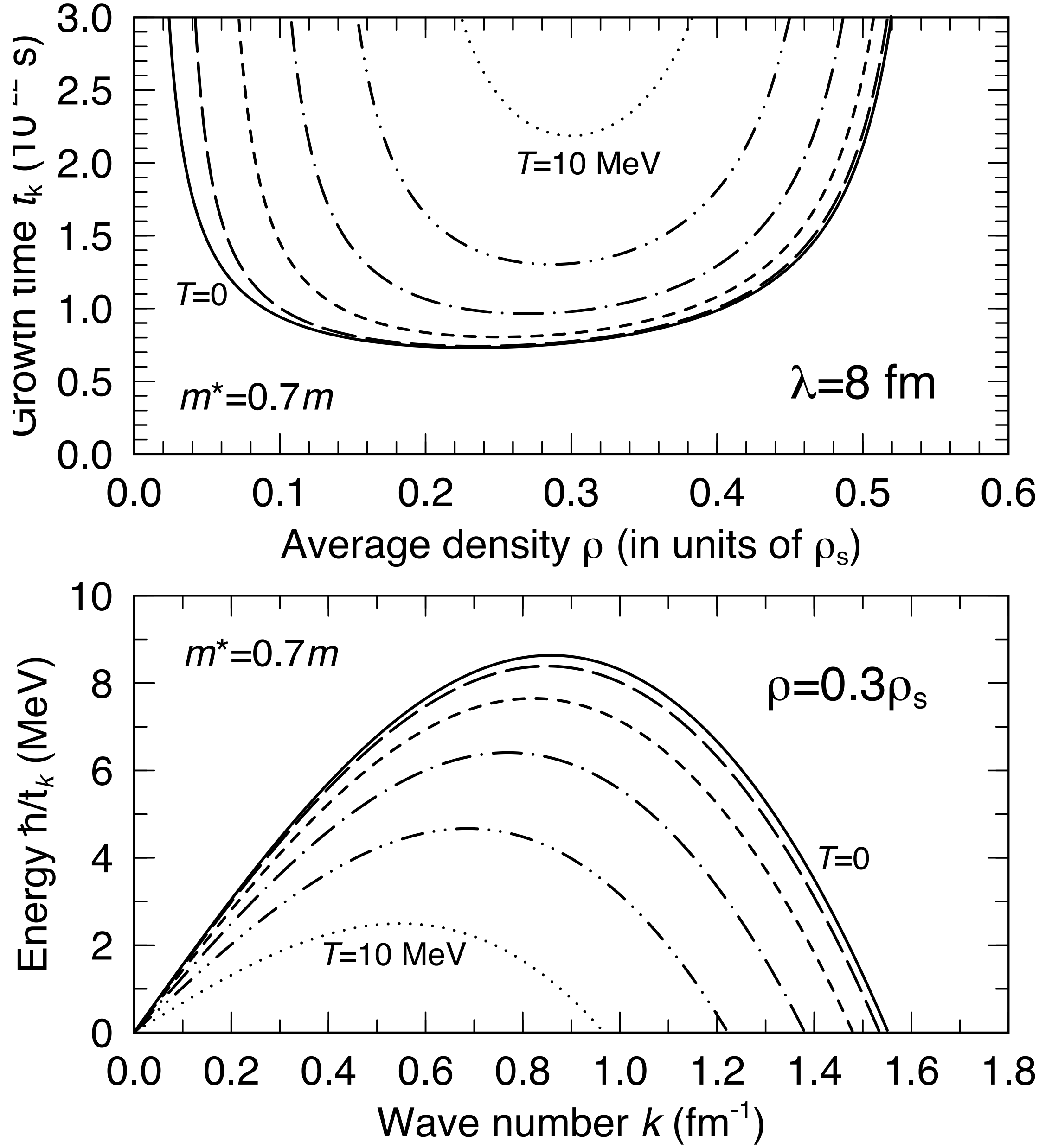
Spinodal boundary
for given wave length λ



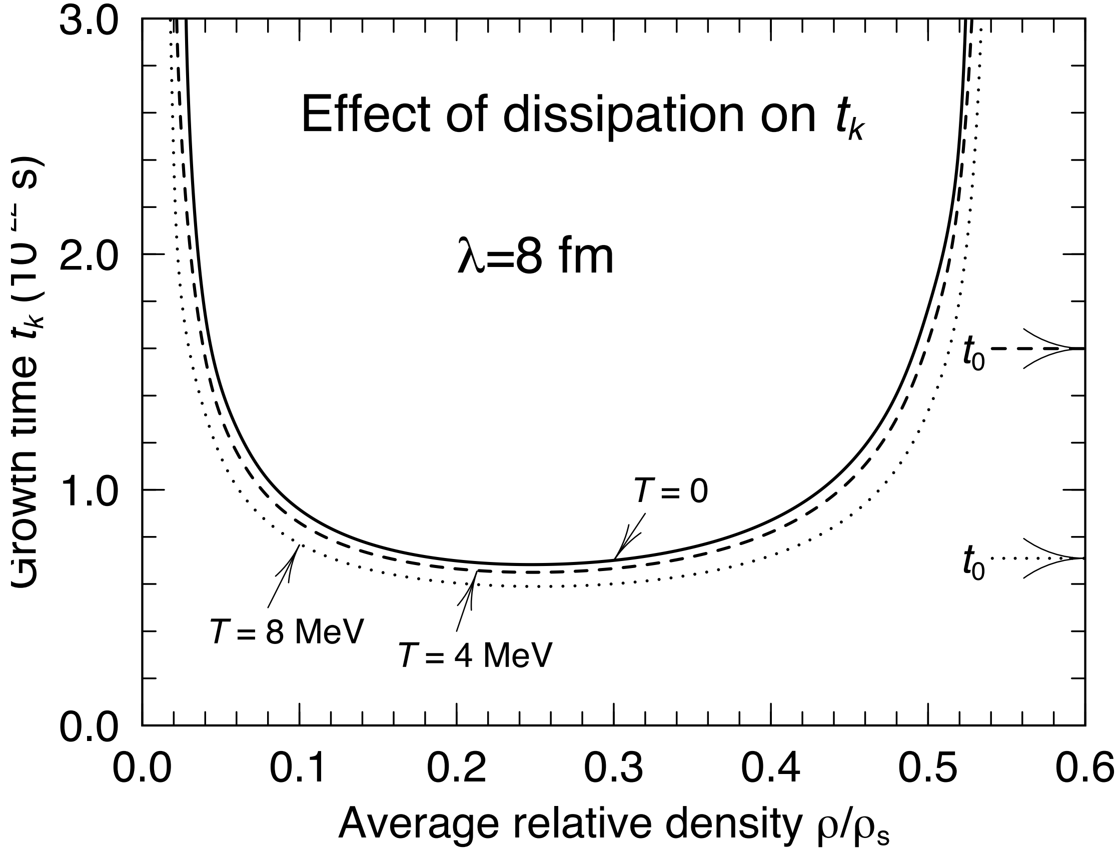
Growth times t_λ for λ = 8 fm



Dependence of growth rates on density, temperature and wave length:

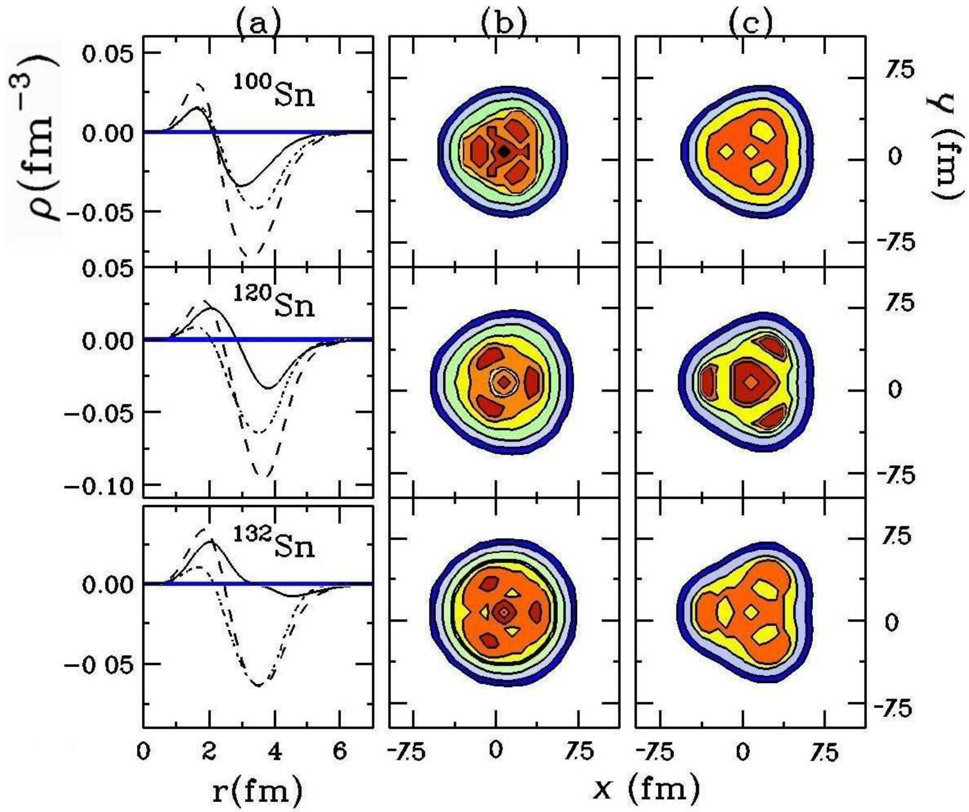


Effect of dissipation



The effect of the growth times t_k from the BUU collision term calculated in the relaxation-time approximation using $t_0(T)$.

Spinodal instabilities in finite nuclear systems



RPA calculations for unstable octupole modes in Sn isotopes:

(a) radial dependence of the form factor at the dilution D = 1:5 for neutrons (solid), protons (dotted), and nucleons (dashed);

(b) contour plots of the perturbed neutron density;

(c) contour plots of the perturbed proton density.

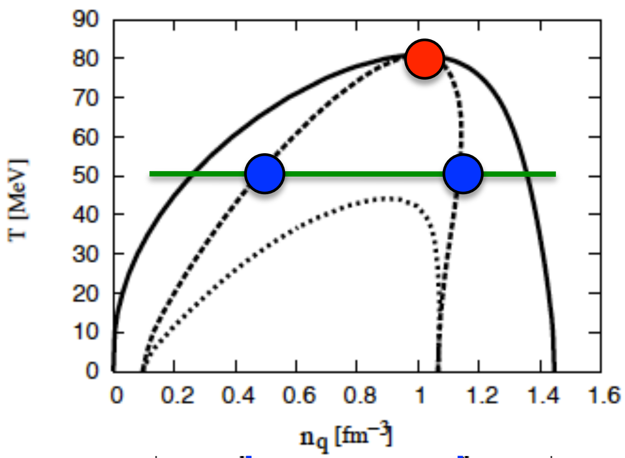
M. Colonna, Ph. Chomaz, S. Ayik, Phys. Rev. Lett. 88 (2002) 122701

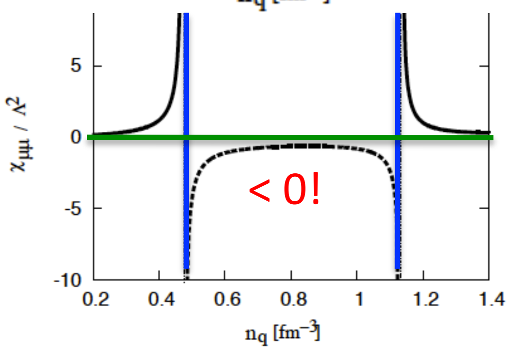
Density fluctuations in the presence of spinodal instabilities

C. Sasaki, B. Friman, K. Redlich, Phys. Rev. Lett. 99, 232301 (2007)

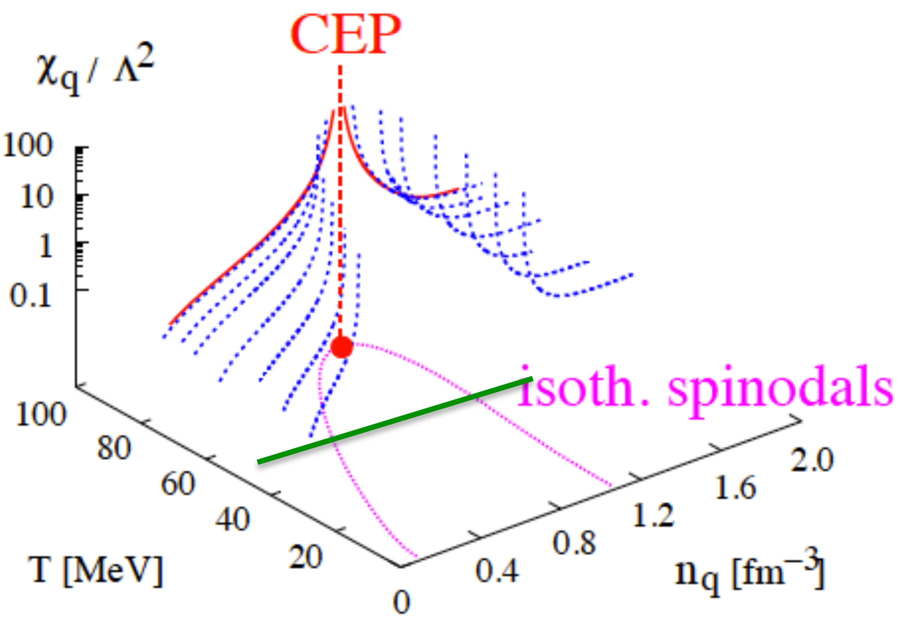
Nambu – Jona-Lasino model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + \mu\gamma_0)\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\vec{\tau}\gamma_5\psi)^2\right]$$





Net quark number susceptibility at T=50 MeV as a function of the quark number density across the first-order phase transition



The net quark number susceptibility
In the stable and meta-stable regions

Dynamics of collective modes in many-body systems

Amplitude evolution: $\omega_\nu = \epsilon_\nu + i\gamma_\nu$

$$\frac{d}{dt}A_\nu(t) \;=\; -i\omega_\nu A_\nu(t) \;+\; B_\nu(t)$$

Correlation function:

$$\sigma_{\nu\mu}(t_1,t_2) \;\equiv\; \prec A_\nu(t_1)\, A_\mu(t_2)^* \succ$$

Markovian noise:

$$\prec B_\nu(t)\, B_\mu(t')^* \succ \;=\; 2\mathcal{D}_{\nu\mu}\,\delta(t-t')$$

Evolution: *seed* *feedback*

$$\frac{d}{dt}\sigma_{\nu\mu}(t) \;=\; 2\mathcal{D}_{\nu\mu} \;-\; i(\omega_\nu - \omega_\mu^*)\sigma_{\nu\mu}$$

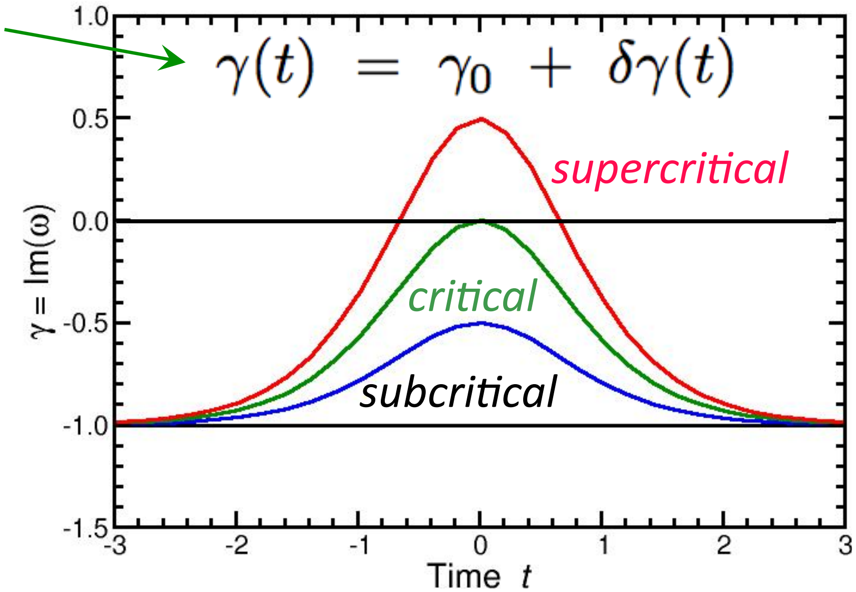
Variance of a single mode:

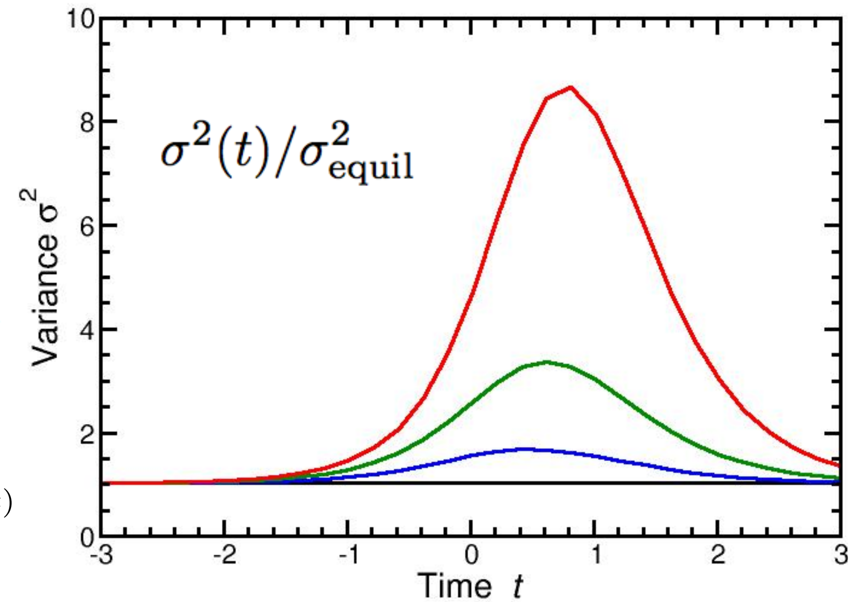
$$\frac{d}{dt}\sigma_\nu^2 \;=\; 2\mathcal{D}_\nu \;+\; 2\gamma_\nu\sigma_\nu^2 \qquad \Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

=> $$\sigma_\nu^2(t) \;=\; \left[2\mathcal{D}_\nu \int_0^t \mathrm{e}^{-2\Gamma_\nu(t')}dt' + \sigma_0^2\right]\mathrm{e}^{2\Gamma_\nu(t)}$$

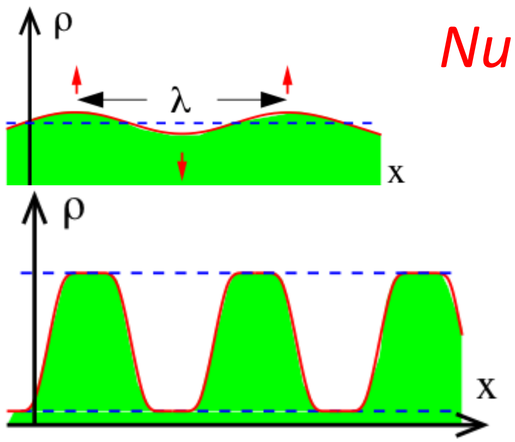
$$\gamma_\nu < 0: \quad \sigma_\nu^2(t) \;\to\; -\mathcal{D}_\nu/\gamma_\nu$$

#4



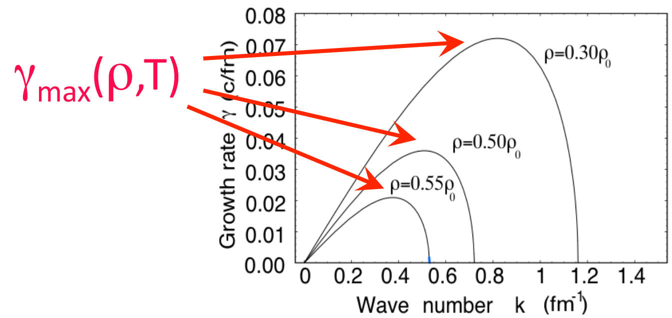


Nuclear spinodal fragmentation

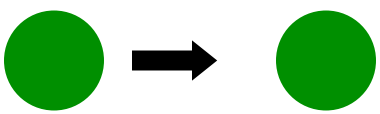


Spinodal pattern

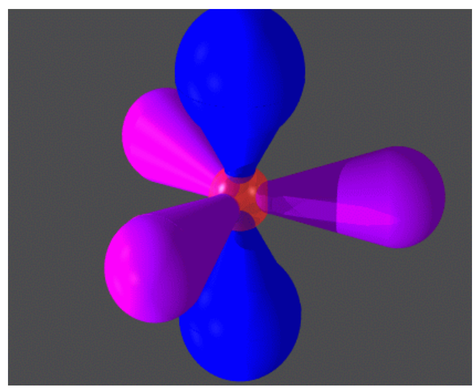
Growth *rate* depends on λ:



The fastest mode becomes dominant!



The massive fragments become nearly *equal*!

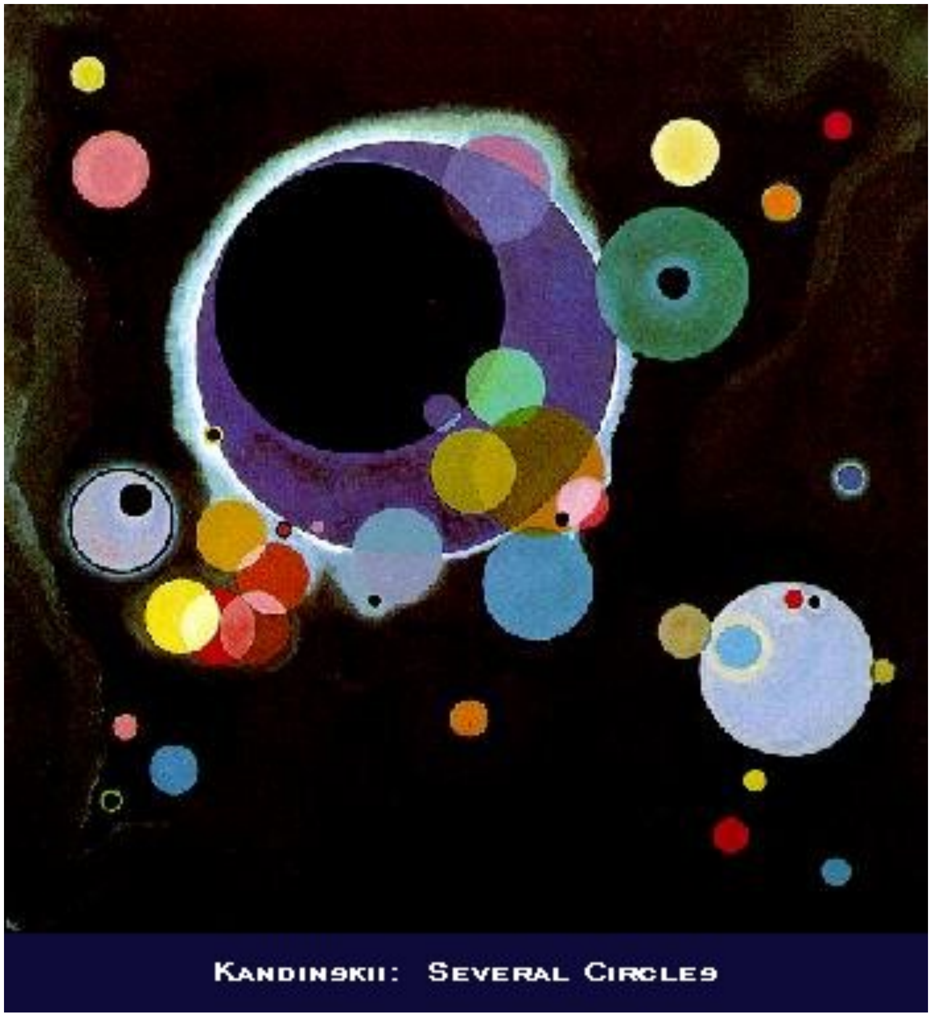


Highly <u>non</u>-statistical =>
<u>Good</u> candidate signature

Identification does not need modeling: "Easy to look for"

Chomaz, Colonna, Randrup: *Nuclear Spinodal Fragmentation*, Physics Reports 389 (2004) 263

Statistical multifragmentation:



=> *Different* fragment sizes

(Igor Mishustin)

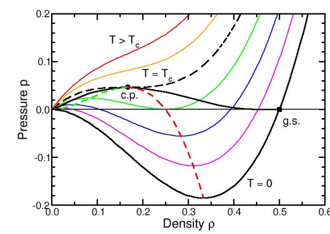
Spinodal fragmentation:

=> *Equal* sizes

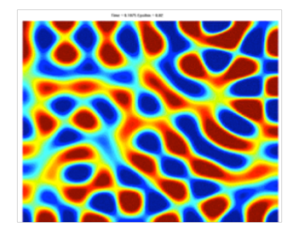
The nuclear liquid-gas phase transition

revealed by collective dynamics in energetic nuclear collisions

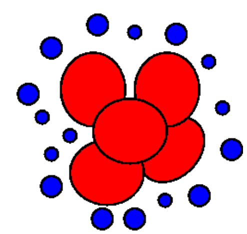
✓ • Thermodynamics: Phase coexistence



✓ • Spinodal instability: Dispersion relations



• Transport simulation: Spinodal fragmentation



Thermodynamics *versus* *Nuclear Collisions*

large, uniform ⟷ small, non-uniform

static, equilibrium ⟷ dynamic, non-equilibrium

large & old

?

Au → Au

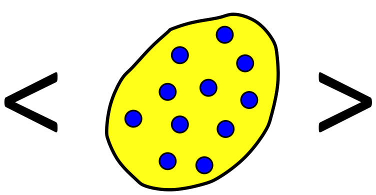
small & young

Dynamical models are indispensable!

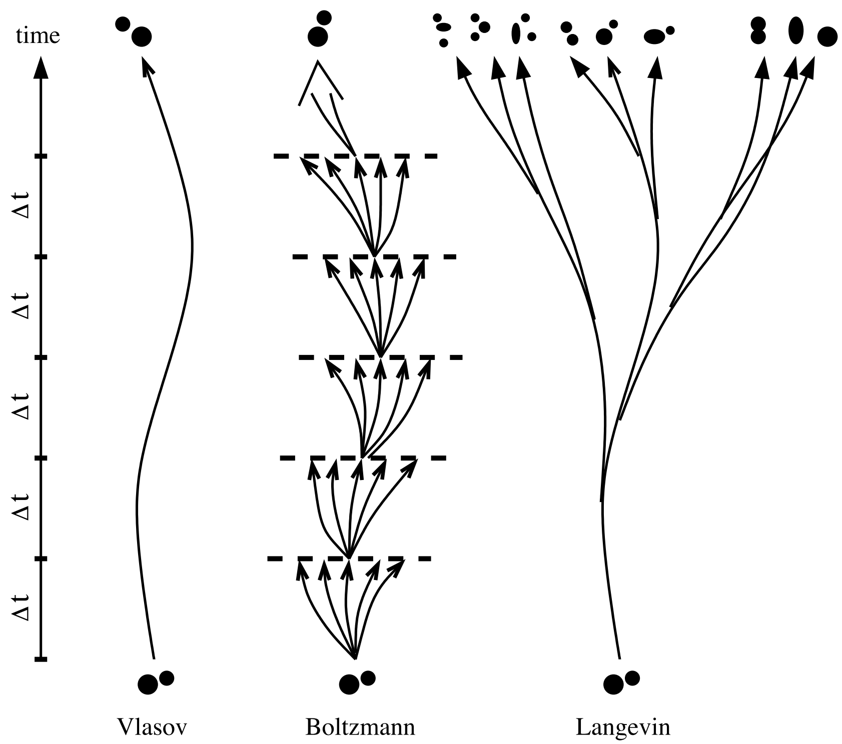
Nuclear Boltzmann-Langevin transport model

Equation of motion: $\dot{f} \;\equiv\; \partial_t f - \{h[f], f\} \;\doteq\; C[f] \;=\; \bar{C}[f] + \delta C[f]$

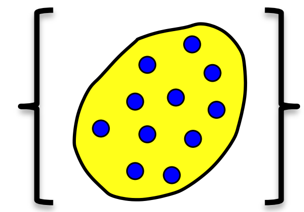
for the one-particle phase-space density: $f(r, p, t)$



average

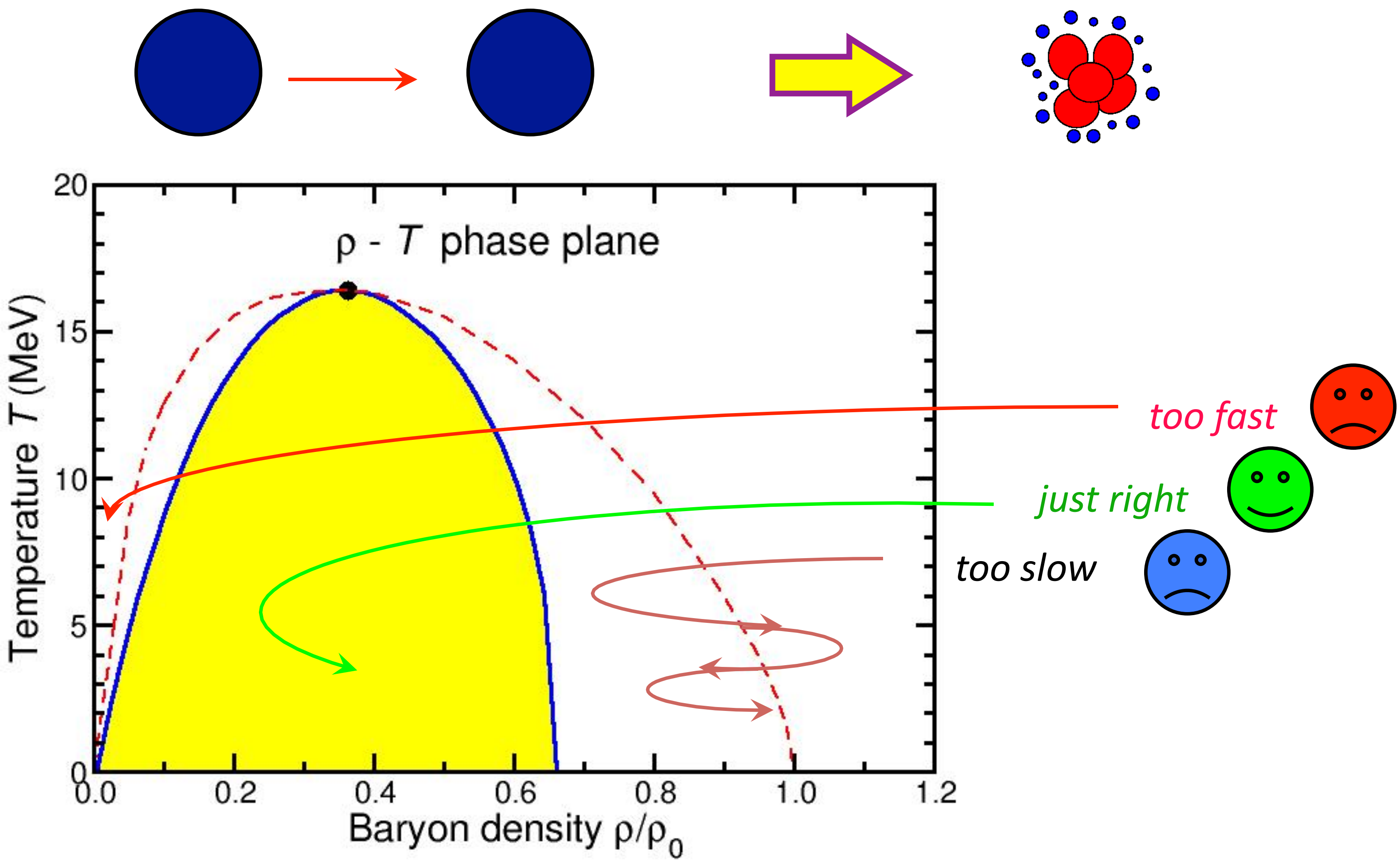


$C = 0$ $\quad C = \bar{C}$ $\quad C = \bar{C} + \delta C$



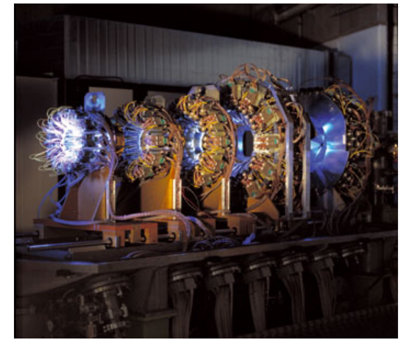
ensemble

Optimal collision energy



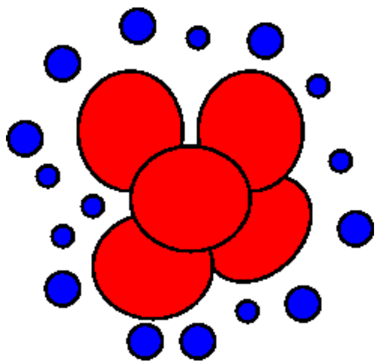
Experiment: *INDRA @ GANIL*

B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252



INDRA

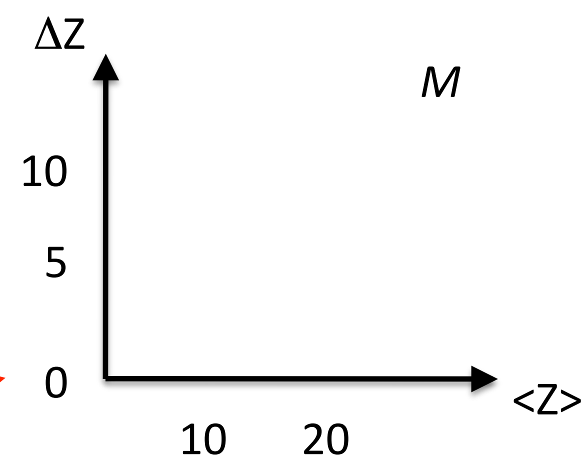
32 MeV/A Xe + Sn (b=0)



Analysis:

For each event having *M* IMFs, calculate mean IMF charge <Z> and IMF charge dispersion ΔZ. (L.G. Moretto)

Make LEGO plot of (<Z>, ΔZ):



For events with ΔZ=0, all *M* IMFs have the same charge

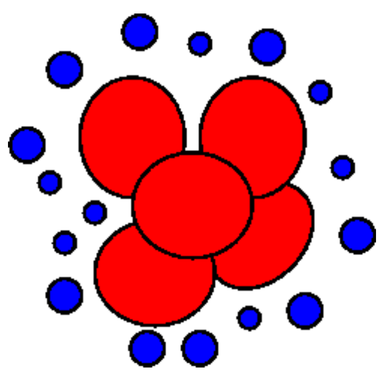
Transport calculations

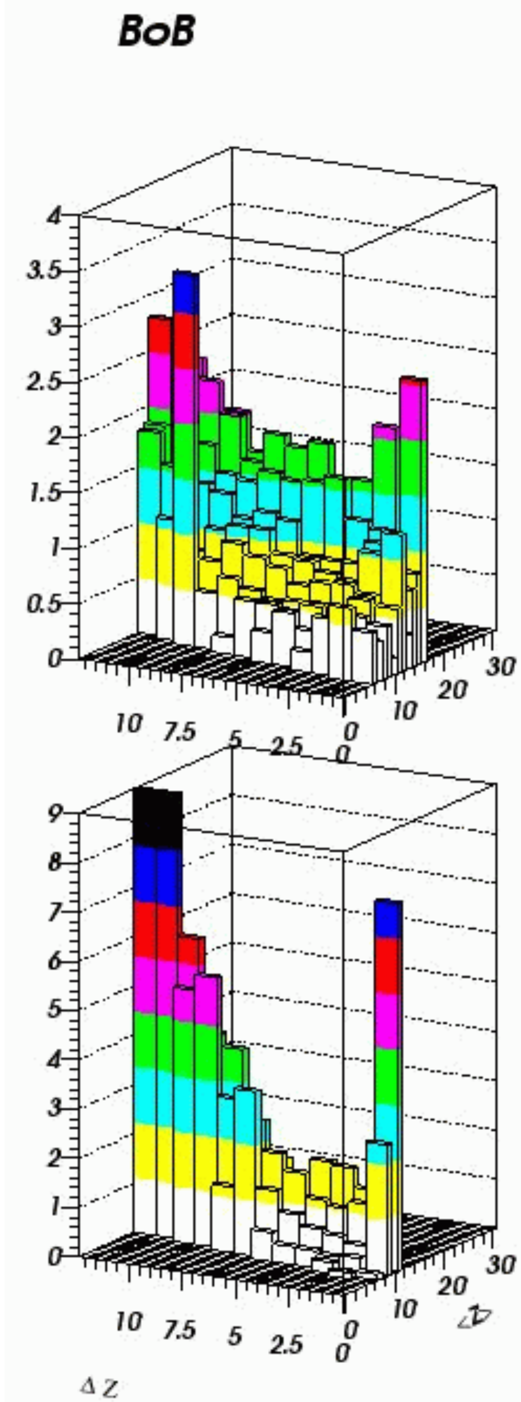
… suggest a visible spinodal signal:

Brownian One-Body dynamics *)
≈ Boltzmann-Langevin

$$\delta K[f] \;\rightarrow\; -\delta \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}}$$

32 MeV/A Xe + Sn (b=0):





M = 4

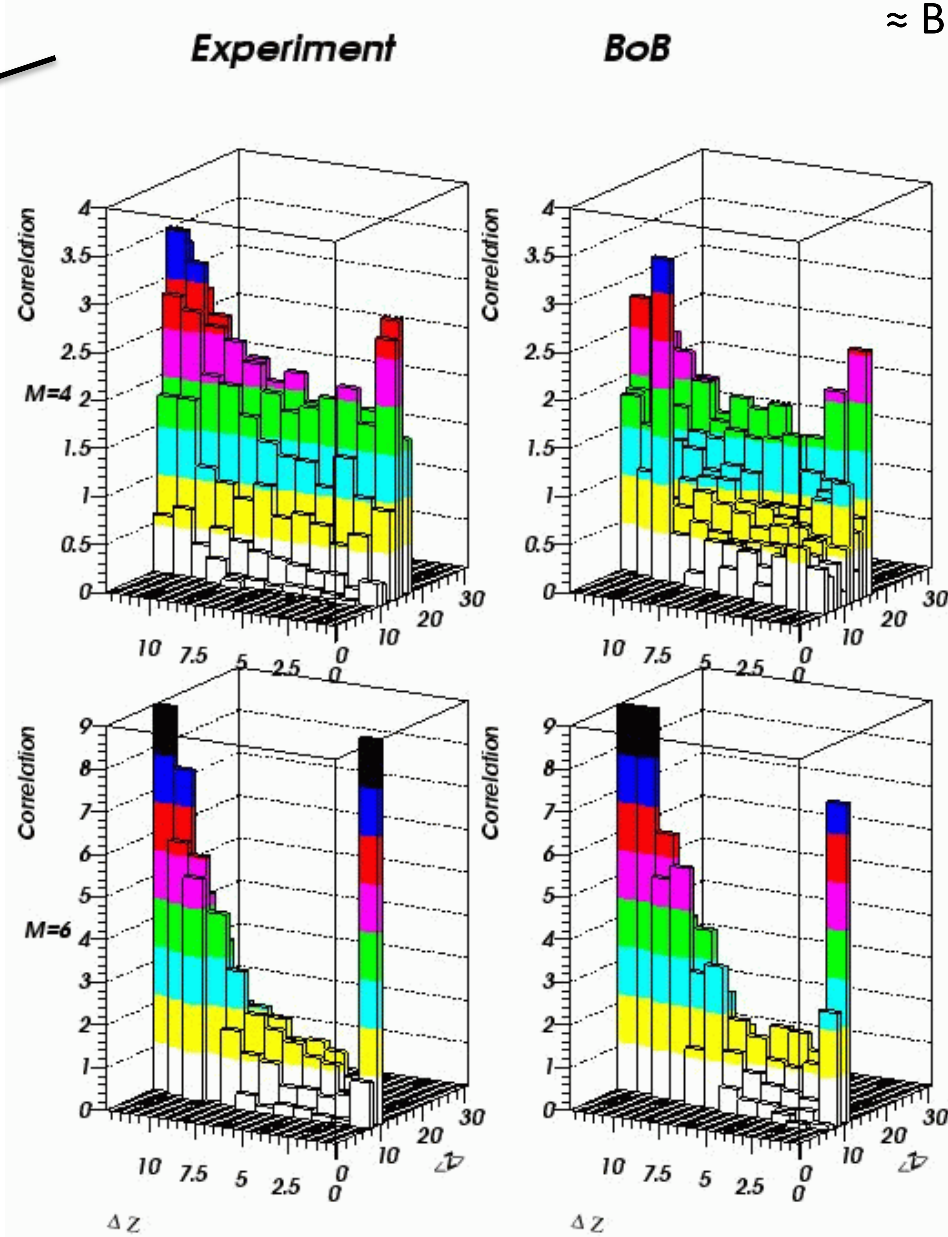
M = 6

*) Ph. Chomaz, M. Colonna, A. Guarnera, J. Randrup, Physical Review Letters 73 (1994) 3512

$$\delta K[f] \rightarrow -\delta \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}}$$

Brownian One-Body dynamics
≈ Boltzmann-Langevin

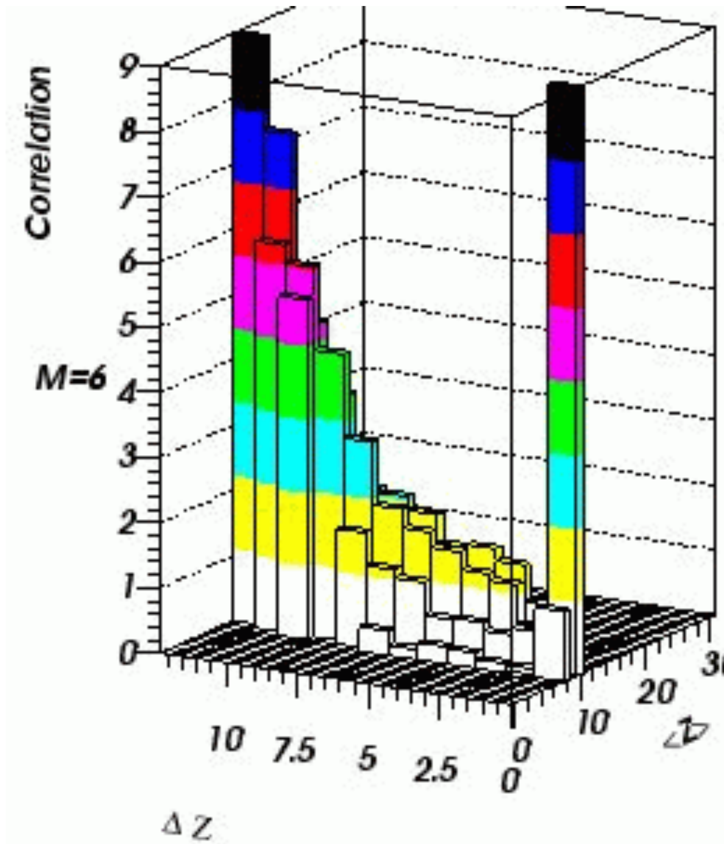
Experiment: *INDRA @ GANIL*



B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252

Ph. Chomaz *et al*, Phys. Rev. Lett. 73 (1994) 3512

Spinodal phase separation does occur for the liquid-gas transition:



Does spinodal phase separation occur for the confinement transition?